Discrete Case of the GBM For the Standard Experiment:

Testing the MM experimentally involves by necessity some stylizing. In an experiment, strategy spaces are usually discrete (in tokens). [1] **Here we derive the GBM's pure strategy equilibria for the discrete case of our experimental design where N = 12, n = 4, w = 100, and m is either 0.5 or 0.3. The set of feasible strategies is restricted to the set of integers from 0 to 100.**

In the discrete (experimental) case, some additional low-efficiency equilibria emerge (as compared to the continuous case). In these additional equilibria some subjects contribute low amounts and the rest contribute nothing. The additional, payoff equilibria are clearly not chosen by the subjects but are treated here for the sake of completeness. We enclose them here only to give the reader insight of the differences between the continuous and discrete cases.

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We used two methods to find the equilibria in the discrete cases of the experiment. First, we conducted an exhaustive brute-force search (via computer simulations) of all strategy combinations possible. Second, we analytically eliminated all configurations that cannot be equilibria. The computer program is available from the authors upon request and the formal elimination of equilibrium candidates follows here below. The results of the two approaches are the same.

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[1] Note that many of the team contributions relevant to a meritocracy, such as effort, may be better characterized as continuous rather than discrete.

Analysis of the discrete case where $N = 12$, $n = 4$, w = 100, and m is either 0.5 or 0.3

First, $s_1 = s_2 = \ldots = s_{12} = 0$ is an obviously equilibrium in the discrete case as well: since $m < 1$, no player can profit from contributing a strictly positive amount to the group account if all others contribute zero. Further, note that the strategy combinations in which b players contribute fully and N-b players contribute nothing to the group account with $b = 10$ (for $m = 0.5$) and $b = 9$ (for $m = 0.3$) are also equilibria in the discrete case: since there are no strategies that allow profitable deviations in the continuous case, there are none in the discrete case either, in which the set of feasible strategies is a strict subset of the set of strategies in the continuous case. In what follows, we analyze how the discreteness of our experimental design changes the equilibrium: Discreteness leads to some additional low-efficiency equilibria in which some subjects make very low contributions while the rest contribute nothing.

Consider now the case in which some players make strictly positive contributions l_1. Moreover, assume that there is no other player contributing more than l_1 to the group account. Let L_1 be the set of players contributing l_1 (i.e. $s_i = l_1 \; \forall \; i \in L_1$), and $|L_1|$ the number of players contributing l_1. As in the continuous case, $|L_1| < N$, else each player would profit from unilaterally changing his contributing from l_1 to zero.

1. Given the parameters of the experiment, we show that Observation 2 of the paper holds as well (even though it does not in a general discrete case).

Assume to the contrary that ($|L_1|$ mod n) = 0. Then, the second highest contribution is $l_1 - 1$, else each high contributor would have an incentive to deviate to $l_1 - 1$, which would not change her group membership. The payoff of the player who contributes $l_1 - 1$ is at most

$$100 - l_1 + 1 + m\delta$$

where δ denotes the total contribution to the group account of the mixed group with contributions of l_1 and l_1 -1.. If this player would change her contribution to l_1 her payoff becomes

$$100 - l_1 + \frac{1}{N - |L_1| + 1} m(\delta + 1) + \frac{N - |L_1|}{N - |L_1| + 1} mnl_1 \ .$$

Thus, a necessary equilibrium condition is

$$\begin{aligned}
&100 - l_1 + 1 + m\delta \geq 100 - l_1 + \frac{1}{N - |L_1| + 1} m(\delta + 1) + \frac{N - |L_1|}{N - |L_1| + 1} mnl_1 \\
&\Leftrightarrow 1 + \frac{N - |L_1|}{N - |L_1| + 1} m\delta \geq \frac{1}{N - |L_1| + 1} m + \frac{N - |L_1|}{N - |L_1| + 1} mnl_1 \\
&\Leftrightarrow N - |L_1| + 1 + (N - |L_1|) m\delta \geq m + (N - |L_1|) mnl_1
\end{aligned}$$

Since $\delta \leq n\ (l_1 - 1)$, one obtains

$$\begin{aligned}
&\Leftrightarrow N - |L_1| + 1 + (N - |L_1|) mn(l_1 - 1) \geq m + (N - |L_1|) mnl_1 \\
&\Leftrightarrow N - |L_1| + 1 - (N - |L_1| \ mn) \geq m \\
&\Leftrightarrow m \leq \frac{N - |L_1| + 1}{n(N - L_1) + 1}
\end{aligned}$$

In our experiment ($|L_1|$ mod n) = 0 if either $L_1 = 4$ or $L_1 = 8$. This translates to $m < 9/33$ or $m <$ 5/17, respectively, and none of these requirements holds for $m = 0.5$ or for m = 0.3.

2. Lemma 1 in the paper also needs to be adjusted for the discrete case. Unlike under the continuous case $|L_1|$ *mod* $n > 0$ does not imply that the highest contribution must be w in equilibrium. This happens because, in order to be classified in the group with only high contributors, a player has to increase his contribution by 1 instead of ε. Similarly, a player who is contributing 0 has to increase his contribution by 1 to classify in a group with higher contributors. Here we will look at cases in which some players contribute 0 and others contribute a positive amount $l_1 < w$. Let $|L_1|$ *mod* $4 = z$ and $|L_1| < N$. There can be an equilibrium in this case when $\pi_1(l_1) \geq \pi_1(l_1 + 1)$, $\pi_1(l_1) \geq \pi_1(0)$, $\pi_0(0) \geq \pi_0(1)$, and

$\pi_0(0) \geq \pi_0(l_1)$.[2] The inequalities guarantee that none of the players has an incentive to deviate unilaterally from the current strategy. Similar logic applies when $|L_1| = N\text{-}n\text{+}z$ = 8+z but the inequalities are a little different. We have less restrictions here because the 0 contributors are in the lowest group for certainty and they cannot increase their payoff by increasing their contribution by 1. Plugging our specific MPCRs, all possible values for z and l_1 in the inequalities above yields the equilibria shown in Table 1 of this Appendix.

MPCR	**Strategy Profile** (s_{12}, s_1, …, s_1)
0.3	(0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1)
	(0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2)
	(0, 0, 0, 0, 0, 0, 0, 3, 3, 3, 3, 3)
	(0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1)
	(0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2)
	(0, 0, 0, 3, 3, 3, 3, 3, 3, 3, 3, 3)
	(0, 0, 0, 4, 4, 4, 4, 4, 4, 4, 4, 4)
	(0, 0, 0, 5, 5, 5, 5, 5, 5, 5, 5, 5)
	(0, 0, 0, 6, 6, 6, 6, 6, 6, 6, 6, 6)
	(0, 0, 0, 7, 7, 7, 7, 7, 7, 7, 7, 7)
0.5	(0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
	(0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2)

Table 1-Appx.

3. We know from Observation 2 in the paper that in the continuous case $|L_1| > n$ is required in any equilibrium containing positive strategies. Any of the highest contributors can slightly decrease their contribution and still remain in the same group. In the discrete case this is not readily obvious. Let us look at a few different cases here. Let $|L_1| \leq 4$, l_2 be the second highest contribution and $|L_2|$ the number of players contributing l_2, l_3 be the third highest contribution and $|L_3|$ the number of players contributing l_3, etc. Clearly $l_2 = l_1 - 1$ otherwise any player contributing l_1 can decrease his contribution by 1 and stay in the same group.

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[2] To simplify notation we use $\pi_1(l_1)$ instead of $\pi_{l_1}(l_1)$, which is the expected pay-off of a player contributing l_1.

3.1 (| L$_1$| + | L$_2$|) = *4* ⇔ $l_3 = l_2 - 1$ otherwise a player contributing l_2 can decrease his contribution by 1 and stay in the same group. However, this implies that any player contributing l_1 has an incentive to deviate and therefore the case is not a possible equilibrium configuration.

3.2 (| L$_1$| + | L$_2$|) = 8 ⇔ $l_3 = 0$ because otherwise any player contributing l_3 has an incentive to decrease his contribution to 0.

3.2.1. | L_1| = 4 ⇔ l_2 = 1 and l_2=2. In this case equilibrium requires that $\pi_1(1) = w - 1 + 3m \geq w - 2 + 2m + \frac{27}{5}m$, so that the player contributing 1 does not have an incentive to increase his contribution to 2. This implies $m \leq \frac{5}{22}$ ⇔ this cannot be an equilibrium configuration.

3.2.2. | L_1| < 4. Equilibrium requires that

$$\pi_1(l_1) = w - l_1 + ml_1 + m[(z-1)l_1 + (4-z)l_2] \geq$$
$$\pi_1(l_1 - 1) = w - l_1 + 1 + ml_1 - m + m\frac{5-z}{9-z}[(z-1)l_1 + (4-z)l_2] + m\frac{4}{9-z}3l_2 \text{ and}$$

$$\pi_2(l_2) = w - l_2 + ml_2 + m\frac{4-z}{8-z}[zl_1 + (3-z)l_2] + m\frac{4}{8-z}3l_2 \geq$$

$$\pi_2(l_2 + 1) = w - l_2 - 1 + m + ml_2 + m[zl_1 + (3-z)l_2]$$

Combining the inequalities above results in $\frac{5-z}{9-z} \leq \frac{4-z}{8-z}$, which is not possible. Therefore this configuration cannot happen in equilibrium.

3.3 (| L$_1$| + | L$_2$|) =12.

3.3.1 | L_1| = 4 ⇔ players contributing l_2 have an incentive to deviate to 0 ⇔ cannot be an equilibrium.

3.3.2 |L_1| < 4. This case is similar to 3.2.2 above except that the probabilities for staying in certain groups change. Equilibrium requires that

$$\pi_1(l_1) = w - l_1 + ml_1 + m[(z-1)l_1 + (4-z)l_2] \geq$$
$$\pi_1(l_1 - 1) = w - l_1 + 1 + ml_1 - m + m\frac{5-z}{13-z}[(z-1)l_1 + (4-z)l_2] + m\frac{8}{13-z}3l_2$$ and

$$\pi_2(l_2) = w - l_2 + ml_2 + m\frac{4-z}{12-z}[zl_1 + (3-z)l_2] + m\frac{4}{12-z}(n-1)l_2 \geq$$

$$\pi_2(l_2 + 1) = w - l_2 - 1 + m + ml_2 + m[zl_1 + (3-z)l_2]$$

Combining the inequalities above results in $\frac{5-z}{13-z} \leq \frac{4-z}{12-z}$, which is not possible. Therefore this configuration cannot happen in equilibrium.

3.4 (| L$_1$| + | L$_2$|) mod 4 *>0, 8* >(| L$_1$| + | L$_2$|) > *4.*

3.4.1 (| L$_1$| + | L$_2$| + | L$_3$|) = 12. Equilibrium requires that $\pi_2(l_2) \geq \pi_2(l_2 + 1)$, $\pi_2(l_2) \geq \pi_2(0)$ and $\pi_0(0) \geq \pi_0(l_2)$. Plugging in the different MPCR's in the inequalities shows that in our specific experimental design such a configuration cannot happen in equilibrium.

3.4.2 (| L$_1$| + | L$_2$| + | L$_3$|) < 12 and (| L$_1$| + | L$_2$| + | L$_3$|) > 8 ⇔ $l_4 = 0$.
Equilibrium requires that $\pi_2(l_2) \geq \pi_2(l_2 + 1)$, $\pi_3(l_3) \geq \pi_3(l_2)$, $\pi_3(l_3) \geq \pi_3(0)$ and $\pi_0(0) \geq \pi_0(l_3)$. Plugging in the different MPCR's and the

various possible strategies in these inequalities shows that in our specific experimental design such a configuration cannot happen in equilibrium.

3.5 ($|L_1| + |L_2|$) mod 4 >0, ($|L_1| + |L_2|$) $>8 \Leftrightarrow l_3 = 0$. Equilibrium requires that , $\pi_2(l_2) \geq \pi_2(0)$ and $\pi_0(0) \geq \pi_0(l_2)$. Plugging in the different MPCR's in the inequalities shows that in our specific experimental design such a configuration cannot happen in equilibrium.

4. $|L_1|$ *mod 4* > 0, $|L_1| < 8$.

4.1. ($|L_1| + |L_2|$) $< 8 \Leftrightarrow l_3 = l_2 - 1$. This is similar to cases 3.2.2 and 3.3.2 above applied to l_2 and l_3 instead of to l_1 and l_2, and the result is the same. Such a configuration does not exist in equilibrium.

4.2 ($|L_1| + |L_2|$) $= 8 \Leftrightarrow l_3 = l_2 - 1 \Leftrightarrow$ The player contributing l_3 has an incentive to drop his contribution to 0. This cannot be an equilibrium configuration.

4.3 ($|L_1| + |L_2|$) $> 8 \Leftrightarrow l_3 = 0$. Equilibrium requires that $\pi_1(l_1) \geq \pi_1(l_1 + 1)$, $\pi_1(l_1) \geq \pi_1(l_1 - 1)$, $\pi_2(l_2) \geq \pi_2(l_2 + 1)$, $\pi_2(l_2) \geq \pi_2(0)$, $\pi_0(0) \geq \pi_0(l_1)$. If we plug the two different MPCR's in these inequalities, all possible values for l_1 and l_2, we can find more equilibria. These are given in Table 2 of the Appendix.

MPCR	**Strategy Profile** ($s_{12}, s_{11}, \ldots, s_1$)
0.3	(0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 2)
	(0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 2)
	(0, 0, 0, 2, 2, 2, 2, 4, 4, 4, 4, 4)
	(0, 0, 0, 2, 2, 2, 2, 5, 5, 5, 5, 5)

Table 2-Appx.

5. $|L_1| \bmod 4 > 0, |L_1| > 8 \Leftrightarrow l_2 = 0$ which was already described in 2 above.

THIS CONCLUDES THE ANALYSIS.